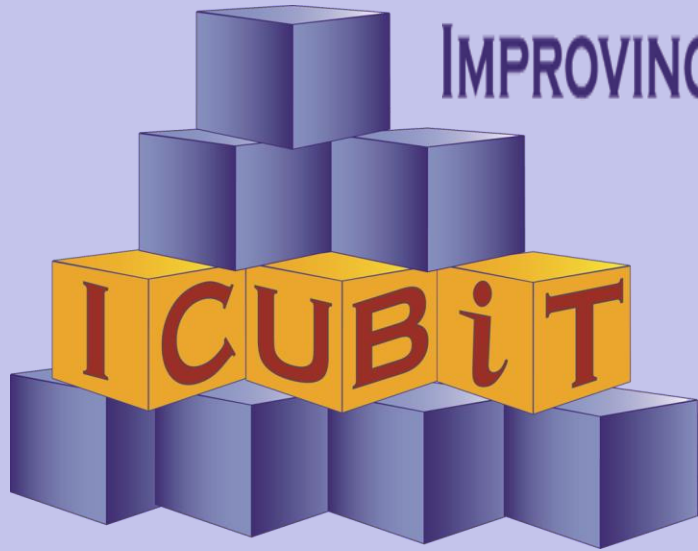


IMPROVING CURRICULUM USE FOR BETTER TEACHING

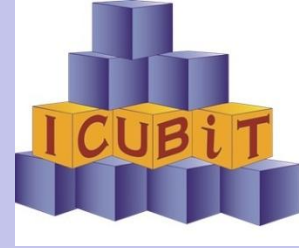


Conceptualizing and Studying Teachers' Curriculum Capacity

April 10, 2011

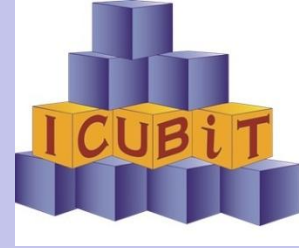
AERA Symposium

About the Project



- NSF Study: Assessing Teachers' Pedagogical Design Capacity and Mathematics Curriculum Use (called ICUBiT)
- PDC: Individual teacher's ability to perceive and mobilize curricular resources in order to design instruction (Brown, 2009)
- Goal:
 - Identify the components of PDC that support curriculum use
 - Develop tools for measuring it

Project Components



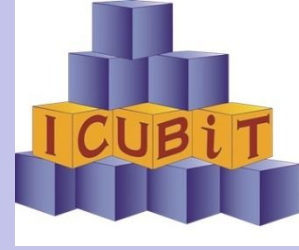
Year 1

- Analyze 5 elementary math curriculum programs
- Develop tool to measure curriculum-embedded mathematics knowledge
- Interview teachers about curriculum use

Year 2

- Refine and field test CEMA
- Develop and refine instrument to measure how teachers *read* and *use* curriculum materials

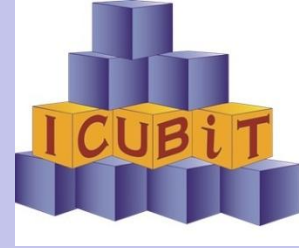
Project Components



Years 3 & 4

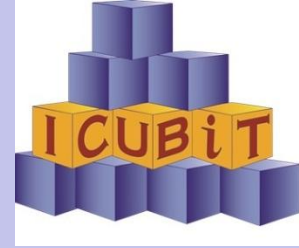
- Use tools to collect and analyze data on how teachers *read* and *use* curriculum materials
- Construct framework to identify, measure, and further develop PDC

Overview of Session



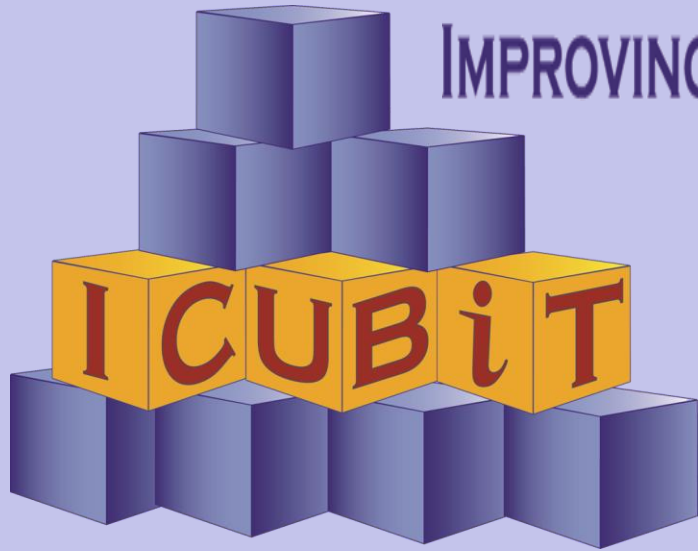
- *A Comparative Analysis of Mathematical and Pedagogical Components of Five Elementary Mathematics Curricula* (Janine Remillard, Shari Lewis, Naphthalin Atanga)
- *Characterizing the Tasks Involved in Teachers' Use of Curriculum* (Luke Reinke, Nina Hoe)
- *Conceptualizing and Assessing Curriculum Embedded Mathematics Knowledge* (Ok-Kyeong Kim)

Discussants



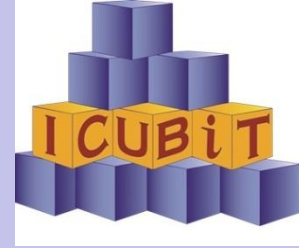
- Andrew Izsák, University of Georgia
- Mary Kay Stein, University of Pittsburgh

IMPROVING CURRICULUM USE FOR BETTER TEACHING



*A Comparative Analysis of
Mathematical and Pedagogical
Components of Five Elementary
Mathematics Curricula*

Curriculum Analysis



- Pedagogical Design Capacity



Curriculum Design

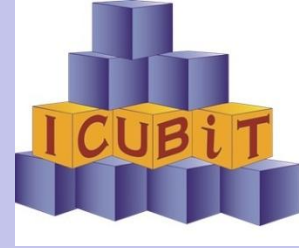
- Questions:

What demands does the curriculum place on teachers?

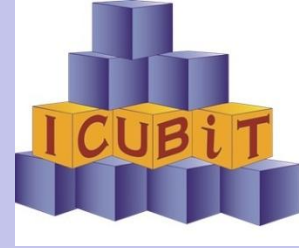
What supports does the curriculum provide the teacher?

Analytical Framework

- Model Lesson (*Imagined Lesson*)
- Voice of the text

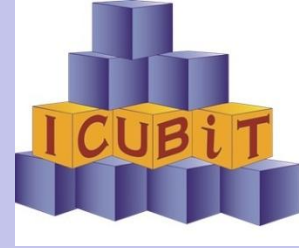


Analytical Framework



- Model Lesson (*Imagined Lesson*)
 - Researcher's model of the author-intended curriculum (lesson level) (Brown, 2008)
 - Mathematical Emphasis
 - Cognitive Demand
 - Key Instructional Representations
 - Instructional Approach (Teacher and student roles)
- Voice of the text

Analytical Framework



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 - Researcher's model of the author-intended curriculum (lesson level) (Brown, 2008)
 - Mathematical Emphasis
 - Cognitive Demand
 - Key Instructional Representations
 - Instructional Approach (Teacher and student roles)
- Voice of the text
 - How the text communicates with the teacher
 - What it communicates about
 - How the text positions the teacher

Five Curriculum Programs

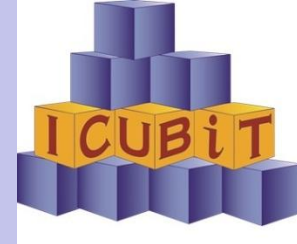
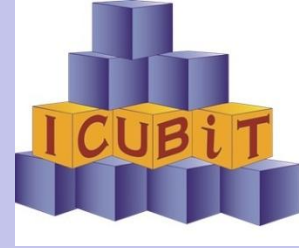


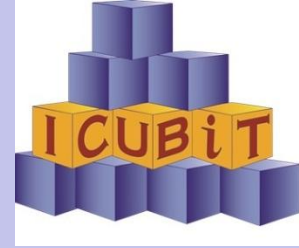
Abb.	Curriculum Title	Developers	Current Publisher
EM	<i>Everyday Mathematics</i> (3 rd Edition)	University of Chicago Mathematics Project	Wright Group/ McGraw-Hill
INV	<i>Investigations in Numbers, Data, and Space</i> (2 nd Edition)	TERC	Pearson
SF	Scott Foresman Mathematics	Scott Foresman/Pearson	Pearson
SM	Primary Mathematics (Standards Editions)	Singapore Ministry of Education	Marshall Cavendish International
TB	Math Trailblazers (3 rd Edition)	TIMS at University of Illinois at Chicago	Kendall Hunt

Methods



- Focus on numbers, operations, Algebra
- Grades 3-5
- Reviewed entire curriculum to understand structure, key features, and emphasis
- Systematically analyzed 3 lessons from each grade (randomly selected)
- Coded for cognitive demand, teacher and student roles, types of communication with the teacher
- Cross-curricular analysis

Cognitive Demand and Teacher's Role



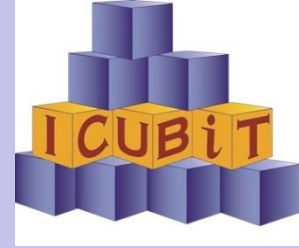
Cognitive Demand

- +Memorization (Mem)
- +Procedures Without Connections (PWOC)
- +Procedures with Connections (PWC)
- +Doing Mathematics (DM)

Role of the Teacher

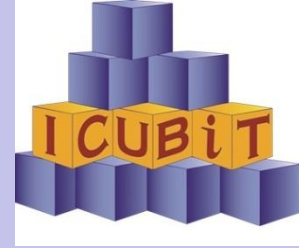
- +What the teacher is expected to do to foster learning

Cognitive Demand



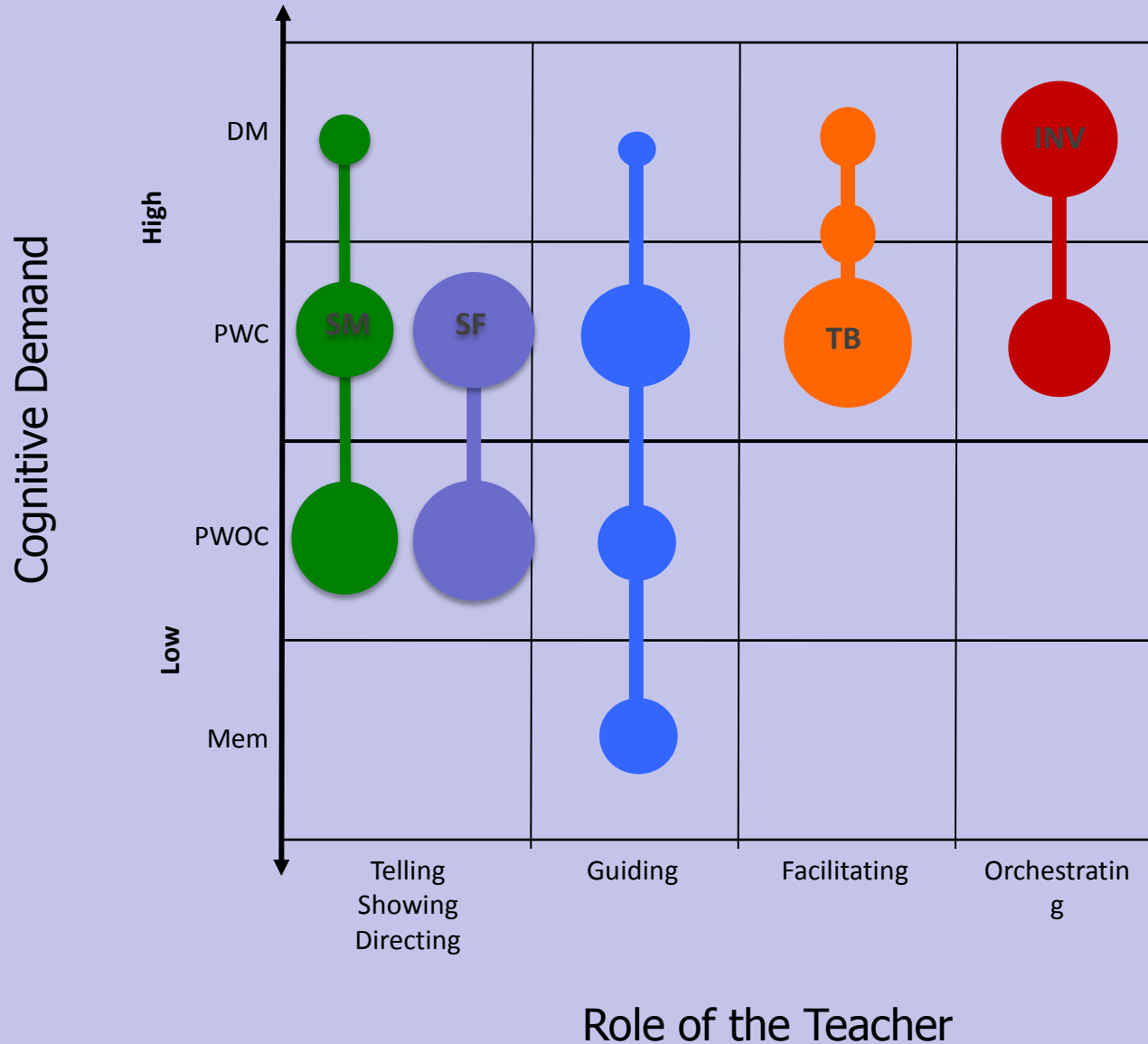
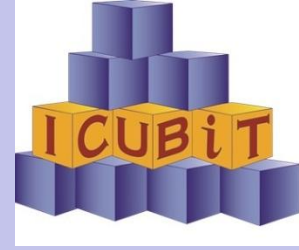
n per curriculum	Memorization	PWOC	PWC	Doing Math
EM n=18	4 (22%)	4 (22%)	9 (50%)	1 (6%)
INV n=11	-	-	5 (45%)	6 (55%)
SF n=18	-	9 (50%)	9 (50%)	-
SM n=21	-	10 (48%)	9 (43%)	2 (9%)
TB n=15	-	-	11 (73%) 2 (13%) PWC/DM	2 (13%)

Teacher's Role

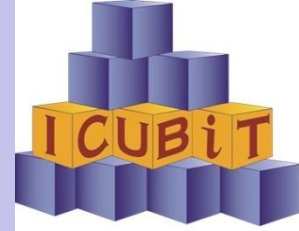


Role	Curriculum
Showing, Telling, Directing	Scott Foresman & Singapore
Guiding	Everyday Math
Facilitating	Trailblazers
Orchestrating	Investigations

Role of the Teacher



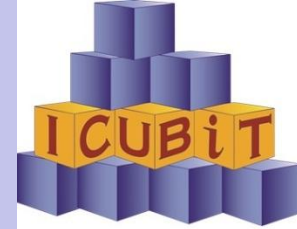
Voice of the Text



Category

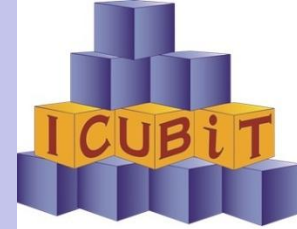
1. Directing Action (Providing Information)
2. Explaining Rationale
3. Anticipating Student Thinking
4. Explaining Math
5. Supporting Teacher Decision Making

Voice of the Text



Type of Support	Examples
Directing Action (providing Information)	<p>Guide students through the subtraction algorithm step-by-step. (SM)</p> <p>Ask children to share other strategies they might use to solve the number story, as you make notes on the board. (EM)</p>
Explaining Rationale	<p>Review the unit box as a way of establishing a real-world context for numbers. (EM)</p> <p>Making representations for these different situations helps students see the actions in each type of problem and how they can use addition and subtraction to solve them. (INV)</p>

Voice of the Text



Type of Support	Examples
Anticipating Student Thinking	<p>Students should understand that the properties justify the steps shown in the three students' papers. (SF)</p> <p>In question 2, a student who understands place value should respond with 40 or 4 tens. (TB)</p>
Explaining Math	<p>Properties of whole numbers explain why you can choose which numbers to multiply first. (SF)</p> <p>The U.S. algorithm for subtraction, sometimes called "borrowing" or the regrouping algorithm, is a procedure that was devised for compactness and efficiency. (INV)</p>
Supporting Teacher Decision Making	<p>A brief review of this lesson's materials may suffice for your class (TB)</p> <p>If you wish, ask children to write a complete sentence to answer the problem. (EM)</p>

Percent of Total Number of Sentences/Phrases Devoted to . . .

	Sentences/ Phrases per Lesson	Directing Action	Explaining Rationale	Anticipating Student thinking	Explaining Math	Supporting Decision Making
EM	116.4	78.6	8.3	7.5	5.6	7.5
		68.2-87.6	5.4-13.9	0-12.9	0.0-18.9	3.4-11.9
INV	114.8	74.3	6.8	12.8	3.9	2.2
		61.8-81.4	1.5-12.3	7.8-23.3	0.0-10.8	0.0-5.5
SF	83.3	86.5	0.5	10.0	3.0	2.2
		81.4-92.0	0.0-2.2	5.3-16.9	1.2-7.8	1.2-4.6
SM	59.8	87.91	1.0	5.2	5.9	0.8
		76.6-95.3	0.0-3.2	0.0-9.7	1.3-13.3	0.0-2.6
TB	128.6	65.5	14.0	13.2	10.5	5.8
		51.1-86.5	8.1-23.1	2.7-25.9	0.0-26.3	1.2-13.1

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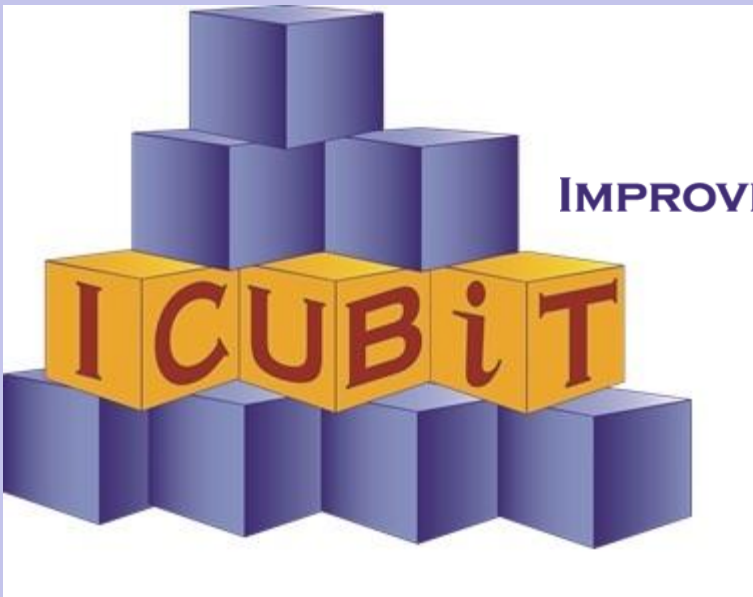
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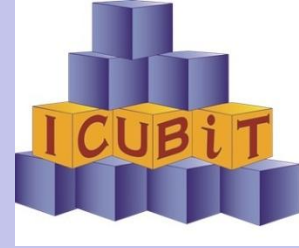


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Characterizing the Tasks Involved in Teachers' Use of Curriculum

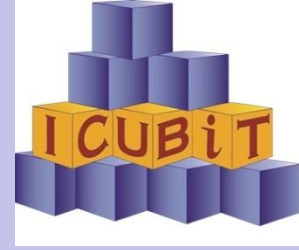
Luke Reinke & Nina Hoe
University of Pennsylvania

Background



- **Two Goals:**
 - Inform the development CEMA (Curriculum Embedded Mathematics Assessment) subscales
 - Further conceptualize Knowledge of Curriculum (Ball, Thames, & Phelps, 2008; Shulman, 1986)

Teachers' Interactions with Curricula

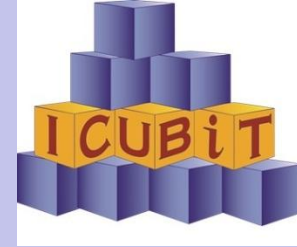


- Curriculum strategy framework (Sherin & Drake, 2008)
 - Categories of interpretive tasks

	Read	Evaluate	Adapt
Before instruction			
During instruction			
After instruction			

- Degrees of appropriation (Brown, 2009)
 - offloading, adapting, improvising

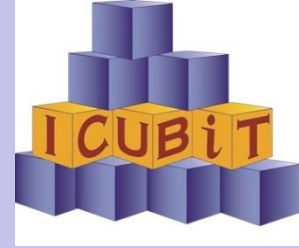
Methods



7 semi-structured interviews

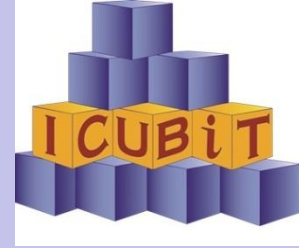
Name	Gender	Years of Experience	Years With Current Curriculum	Current Curricula	Novel Curricula	Grade Levels Taught
Sam	M	7	5	Investigations	Everyday Math	3 rd – 4 th
Alex	F	9	7	Investigations	Everyday Math	2 nd and 4 th
Corey	F	22	1	Scott Foresman	Trailblazers	5 th
Jean	F	32	3	Everyday Math	Investigations	1 st – 4 th
Avery	F	16	6	Investigations	Everyday Math	3 rd – 5 th
Pat	F	12	2	Scott Foresman	Investigations	3 rd
Lee	F	2	2	Houghton-Mifflin	Everyday Math	3 rd

Methods



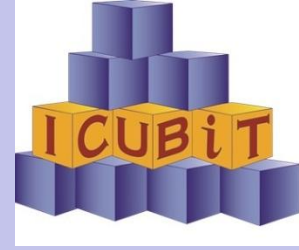
- Identified and coded 63 tasks teachers performed
- Identified 3 categories of tasks:
 - *make sense, evaluate, plan*
- Identified types of knowledge that informed these tasks

Findings



Ways teachers interact with curricula (prior to lesson)	Interpretive Activities (Sherin & Drake, 2008)	Degree of artifact appropriation (Brown, 2009)
make sense	reading	
evaluate	evaluating	
plan	adapting	offloading, adapting, improvising

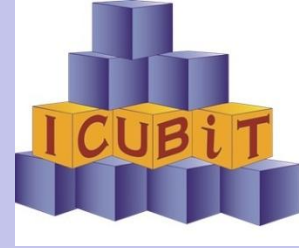
Make Sense



Teachers read to understand

- objective or purpose of the lesson
- activities
- representations, problems, and solution-strategies

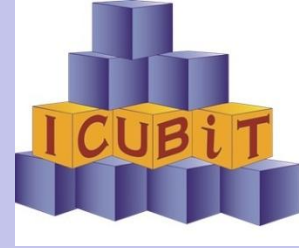
Make Sense



Placing mathematics within larger contexts

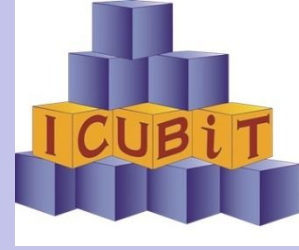
- Connecting different representations of the same mathematical concepts.
- Connecting to other mathematics concepts in the same lesson
- Connecting to other mathematics concepts learned during the year
- Connecting to other mathematics students have or will learn over a lifetime

Evaluate



- Evaluate curricular elements and features with different goals and audiences in mind
 - For *themselves*
 - For *students in general*
 - For *their own particular students*
- Evaluating for students in general and their particular students is closely tied to predicating student responses

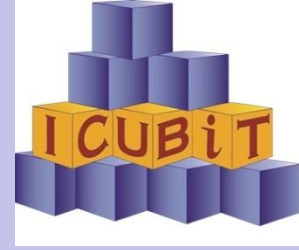
Evaluating for Themselves



“Like I said with Investigations, I love it. I love they way that they think it through, and its kind of the way I thought about math as a child.”

-- Sam

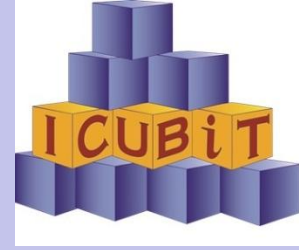
Evaluating for Students in General



“I do like how they make it really clear that 3 times blank means the number of rows. I think it’s important for the kids to know that... 3 ‘X’ is going to tell you 3 groups of blank.”

-- Jean

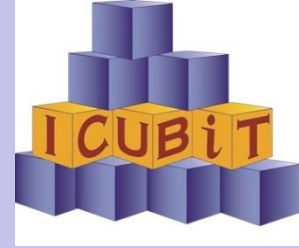
Evaluating for Own Students



“I find this is way too much for my students to comprehend.”

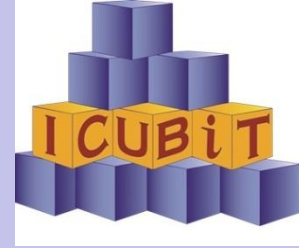
-- Corey

Plan

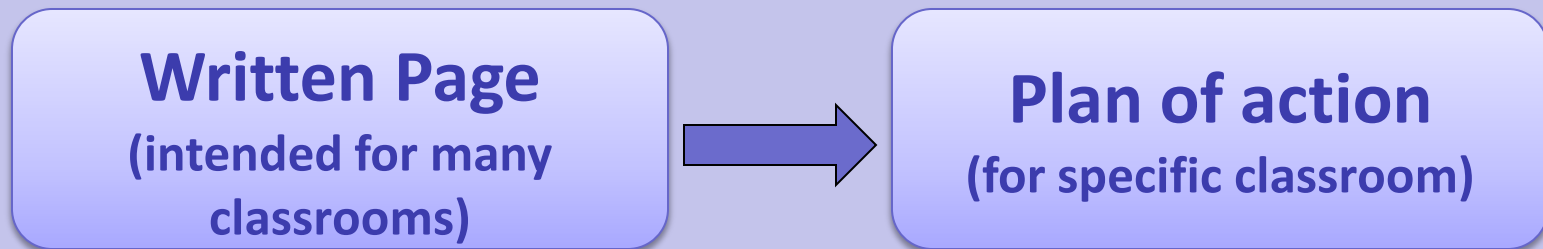


- **Selecting the activities**
- **Transforming the activities as written into a plan for enactment**
 - **Plan activity structure- participation structures, presentation aids, differentiation**
 - **Plan their own actions within an activity**
 - **Plan for student responses within an activity**

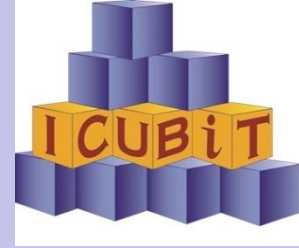
#1: Importance of Predictive Thinking



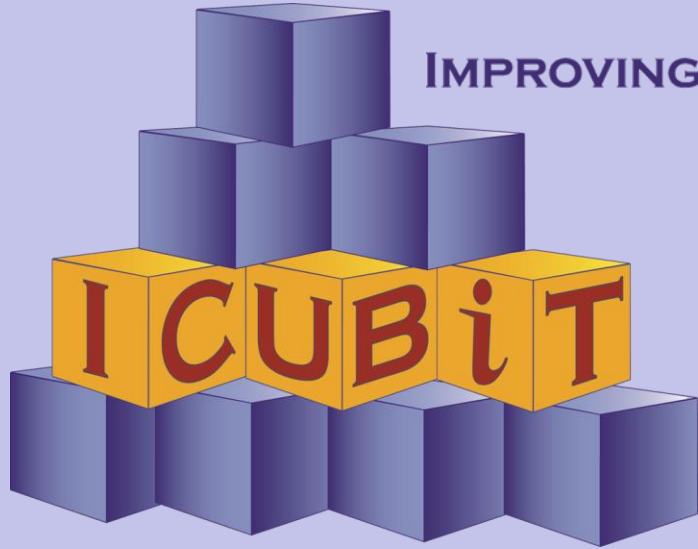
- Teachers rely on knowledge of their classroom context, especially when *evaluating & planning*
- Teachers draw upon past experiences
- Teachers predict results in current classroom
- Transformation from:



#2: Dependent Nature of the 3 Types of Tasks



IMPROVING CURRICULUM USE FOR BETTER TEACHING

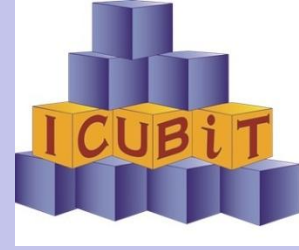


Conceptualizing and Assessing Curriculum Embedded Mathematics Knowledge

Ok-Kyeong Kim

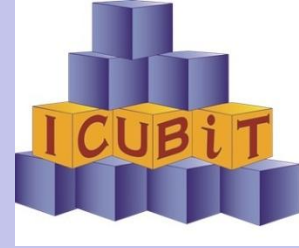
Western Michigan University

Curriculum Embedded Mathematics Knowledge



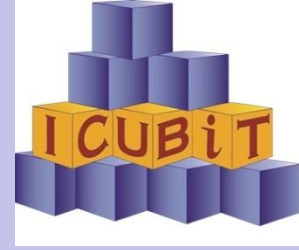
- Mathematics knowledge required to understand the mathematics underlying tasks, instructional designs, and representations in mathematics curriculum materials
- Part of Mathematical Knowledge for Teaching (MKT) by Ball and her group
- Important component of PDC

Our Team



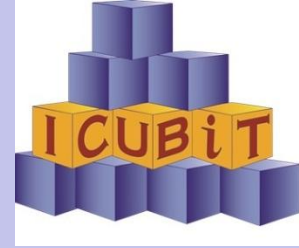
- Two math educators
- One mathematician and math educator
- One psychometrician
- Four research assistants

Curriculum Embedded Mathematics Assessment (CEMA)



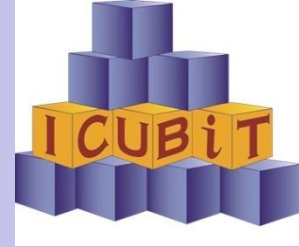
- A prototype of a new tool to measure teachers' understanding of the mathematics embedded in curriculum resources (tasks, representations, teachers' guides, etc.)
- Guiding questions:
 - (1) How are mathematical ideas represented and embedded in various features of elementary curriculum programs?
 - (2) How are these ideas interpreted by elementary teachers?
- Aim: Developing a proof of concept of this specialized knowledge and its relationship to MKT

Curriculum Embedded Mathematics Assessment (CEMA)



- **Structure: Excerpts and associated items**
(8 excerpts, 4-6 questions per excerpt)
- **Five programs used:**
 - *Investigations in Number, Data, and Space*
 - *Everyday Mathematics*
 - *Math Trailblazers*
 - *Scott Foresman Mathematics*
 - *Singapore Mathematics.*
- **Content focus and grade level: Number and operations and algebra strands in grades 3-5.**

CEMA Sample: Excerpt and Questions



[Survey: Longest Multiplication Combination](#)
[Survey: Multiplication Methods](#)
[Survey: Part-Whole Models](#)
[Survey: Baseball Cards](#)
[Demographics Survey](#)

Excerpt

The following is an example that shows two different solution strategies found in a student book.

Baseball Cards Trisha and her brother, Kyle, collect and sell baseball cards. Kyle has 6 cards to sell. Trisha has 3 cards to sell. If they sell the cards for 8¢ each, how much money will they get all together?

Here are two ways to solve the Baseball Cards problem.

Solution 1

Read and Understand

Hidden Question: How many cards did they have to sell all together?

Trisha has 3 cards. Kyle has 6 cards.

$$3 + 6 = 9$$

They had 9 cards to sell.

Question in the Problem: If they sell the cards for 8¢ each, how much money would they get together?

$$9 \text{ cards} \times 8¢ \text{ each} = 72¢$$

Kyle and Trisha will get 72¢ together.

Solution 2

Hidden Question 1: How much money would Trisha get for selling her cards?

$$3 \text{ cards} \times 8¢ \text{ each} = 24¢$$

Hidden Question 2: How much money would Kyle get for selling his cards?

$$6 \text{ cards} \times 8¢ \text{ each} = 48¢$$

Question in the Problem: If they sell the cards for 8¢ each, how much money would they get together?

$$24¢ + 48¢ = 72¢$$

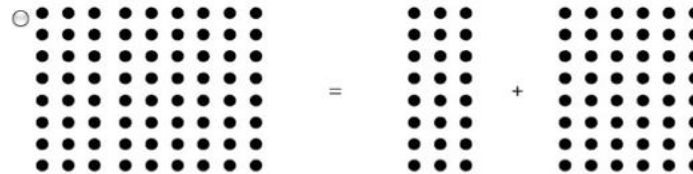
Kyle and Trisha get 72¢ together.

Email Address:
 Email Verification:

Survey: Baseball Cards

Please answer the following questions about the excerpt at the left. Select a single answer for each item unless otherwise specified.

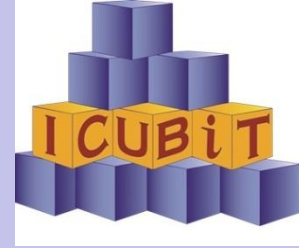
- What fundamental mathematical idea provides the basis for why the two solution methods produce the same answer?
 - Commutative property
 - Relationship between addition and multiplication
 - Distributive property
 - Order of operation
- Select the visual model that best represents the relationship between the two solution strategies in the excerpt.



$\triangle \triangle \triangle + \triangle \triangle \triangle \triangle \triangle \triangle = 9 \text{ cards}$

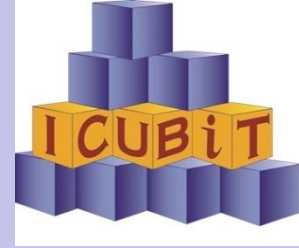


Methods and Procedures



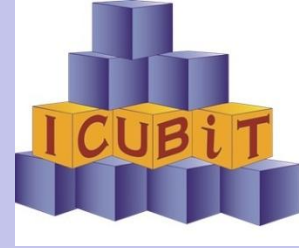
- **Conceptualization**
- **Excerpt and Item Development**
- **Multiple Pilots**
- **Expert Review**
- **Online CEMA Development**
- **Field Test and Item Analysis**

Methods and Procedures



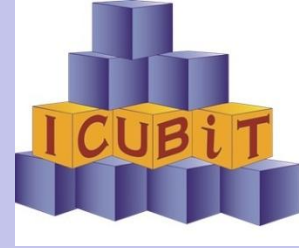
- **Conceptualization (sample questions)**
- **Excerpt and Item Development**
- **Multiple Pilots**
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Conceptualization: Four Dimensions



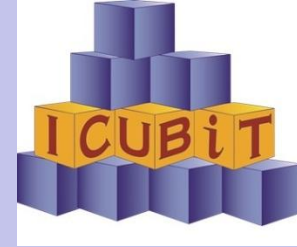
1. **Mathematical ideas** – Knowledge needed to identify the mathematical point of a task/lesson and the mathematical ideas embedded in a task or student work
2. **Surrounding knowledge** – Knowledge of how a particular mathematical goal is situated within a set of ideas, including the foundational and future ideas
3. **Problem complexity** – Knowledge needed to assess relative complexity and difficulty of a variety of mathematical ideas or tasks, and to identify possible points of confusion
4. **Connections across representations** – Knowledge needed to make connections across representations of the same mathematical idea

Conceptualization: We drew on...



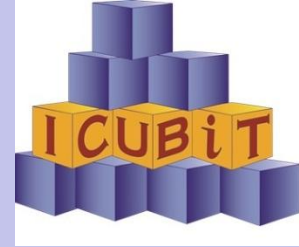
- Curriculum analysis (5 programs)
- Our own experience with elementary teachers and curriculum materials
- Our experience developing preliminary items
- Teacher interviews using curriculum materials
- Literature on teacher knowledge

Conceptualization: Literature Connections



Researcher	Characterization of teacher knowledge					
Shulman's content knowledge in teaching	subject matter content knowledge			pedagogical content knowledge		curricular knowledge
Grossman's pedagogical content knowledge	Pedagogical content knowledge					
				knowledge and beliefs regarding student understanding	knowledge of instructional strategies and representation	conceptions of the purposes of teaching
Ball, et al.'s mathematical knowledge for teaching (MKT)	Subject matter knowledge			Pedagogical content knowledge		
	common content knowledge	knowledge at the mathematical horizon	specialized know knowledge	knowledge of content and students	knowledge of content and teaching	knowledge of content and curriculum
Sleep's mathematical purposing				Mathematical purposing		
				knowledge of content and teaching	knowledge of content and curriculum	

CEMA Sample with Dimensions



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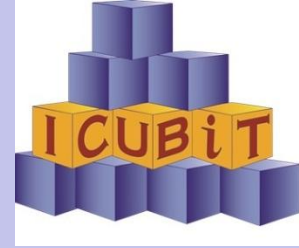
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CEMA Sample with Dimensions



#. What fundamental mathematical idea provides the basis for why the two solution methods produce the same answer?

a) Commutative property

b) Relationship between addition and multiplication

c) **Distributive property**

d) Order of operation

Dimension 1 (Mathematical ideas embedded in the problem)

CEMA Sample with Dimensions

#. Select the visual model that best represents the relationship between the two solution strategies in the excerpt.

a)

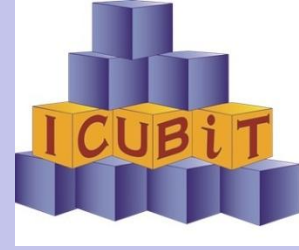
b)

$\triangle \triangle \triangle + \triangle \triangle \triangle \triangle \triangle \triangle \triangle = 9 \text{ cards}$

c)

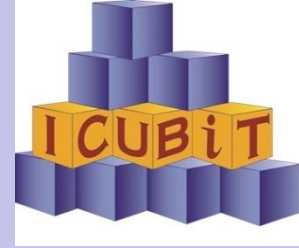
d)

CEMA Sample with Dimensions



#. Write an equation in a generalized form that shows the relationship between the two solutions.

CEMA Sample with Dimensions



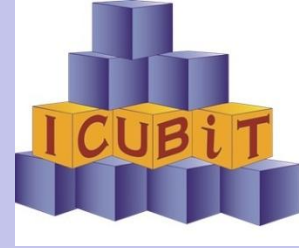
#. Write an equation in a generalized form that shows the relationship between the two solutions

$$(a+b)c = ac + bc$$

Dimension 1 (Mathematical ideas embedded in the problem)

Dimension 4 (Connections across representations)

CEMA Sample with Dimensions



#. Write an equation in a generalized form that shows the relationship between the two solutions

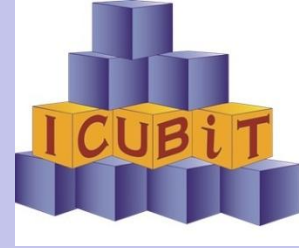
$$(a+b)c = ac + bc$$

Dimension 1 (Mathematical ideas embedded in the problem)

Dimension 2 (Surrounding knowledge)

Dimension 4 (Connections across representations)

CEMA Sample with Dimensions



Below are division problems that students are assigned to model and solve using the method pictured in the excerpt. Order these problems from easiest to most difficult using a scale of 1 to 3. “1” is the easiest to model and solve and “3” is the most difficult.

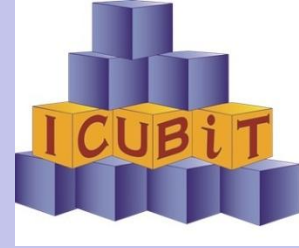
a) $246 \div 6$

b) $179 \div 2$

c) $936 \div 3$

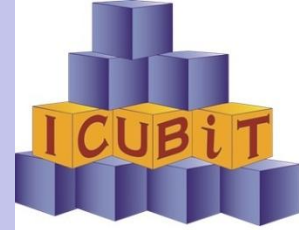
Dimension 3 (Problem complexity)

Issues and Challenges



- **Difficulty of conceptualization and challenges of developing an assessment (repeated process of refinement)**
- **Coordination of mathematical precision, pedagogical importance, and measurement criteria**
- **Scope of the assessment: level of mathematics**

CEMA Sample with Dimensions



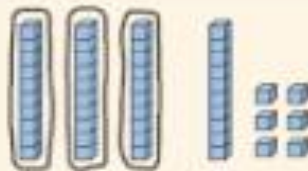
How do you record division?

Example

Find $46 \div 3$.

STEP 1

Divide the tens.

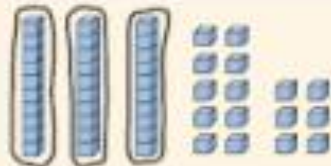


What You Write

$$\begin{array}{r} 1 \\ 3 \overline{)46} \\ -3 \\ \hline 1 \end{array}$$

STEP 2

Regroup by bringing down the ones.



$$\begin{array}{r} 1 \\ 3 \overline{)46} \\ -3 \\ \hline 16 \end{array}$$

STEP 3

Divide the ones.



$$\begin{array}{r} 15 \text{ R}1 \\ 3 \overline{)46} \\ -3 \\ \hline 16 \\ -15 \\ \hline 1 \end{array}$$

$$46 \div 3 = 15 \text{ R}1$$

Summary of Excerpts and Questions

Excerpt	Central mathematical idea	Program	Grade	Number of questions	# of questions in dimensions			
					Dim 1	Dim 2	Dim 3	Dim 4
Place value division	The partitive interpretation of long division algorithm using base-ten blocks	SF	4	6	5	0	1	5
Baseball cards	Distributive property of multiplication over addition	SF	4	4	4	1	0	3
Fact triangles	Commutativity and inverse operations illustrated in fact triangles	EM	3-5	4	3	1	1	1
Multiplication methods	Multi-digit multiplication in partial product method, modified repeated addition method, and traditional algorithm	EM	3-5	6	6	0	1	3
Longest multiplication	Prime factorization and multiples of numbers	INV	5	6	4	3	0	2
Array model	Multiplying by multiples of 10 and its representations	INV	4	4	4	2	0	3
Square numbers	Patterns associated with square numbers	TB	5	6	5	3	2	4
Part whole model	Additive and/or multiplicative number relationships represented by part whole models	SM	4	5	4	0	2	4
Total				41	35	10	7	25